

## PHY 115: Mechanics and Properties of Matter I

(Newton's laws of Motion, Friction, Work and Energy)

### Newton's Laws of Motion

Newton's laws deal with force and motion / acceleration.

#### Newton's First law of Motion

The law states that every object will continue in its state of rest, or of uniform motion in a straight line, unless an external force acts on it. But it is possible to apply a force to an object (i.e. an external force acting on an object) without producing motion. A good example of this is when a car is to be pushed to start the engine by gradually increasing the applied force. It will get to a particular value of the applied force that the car will start to move, that is any values of the applied force below this value is not strong enough to cause the car to move. Therefore it can be said that though an external force is applied, there is no movement or change of state of the car.

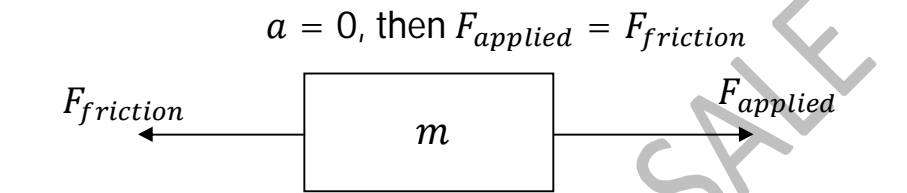


Fig. 1: Applied force is equal to the frictional force between the box of mass  $m$  and the surface, only when acceleration  $a = 0$ .

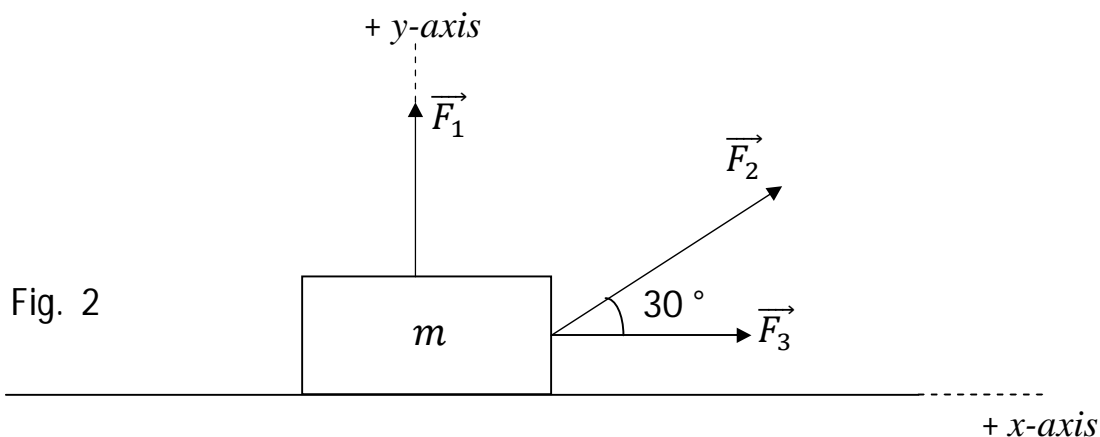
Our basic assumption here, for now, is that there is no friction between the surfaces (i.e. the object and the horizontal plane) in contact. The explanation above is to introduce a new concept called Net Force,  $F_{\text{Net}}$ , to replace the external force  $F_{\text{applied}}$ . Therefore we can state the Newton's first law of motion as follow:

Every object will continue in its state of rest, or of uniform motion in a straight line, unless a net force acts on it. This tendency of an object to resist change of state of motion is called as inertia.

**Inertia:** the resistance an object has to a change in its state of motion.

**If  $F_{\text{Net}} = 0$ , then  $a = 0$  statement of the 1<sup>st</sup> law.**

Consider an object of mass  $m$  under the influence of three forces as shown in Fig. 2 below, the net force  $F_{\text{Net}}$  is the vector sum of all the three forces.



$$\vec{F}_{Net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

Remember that a vector,  $\vec{V}$ , can be expressed as  $\vec{V} = V\cos\theta \, i + V\sin\theta \, j = V_x \, i + V_y \, j$

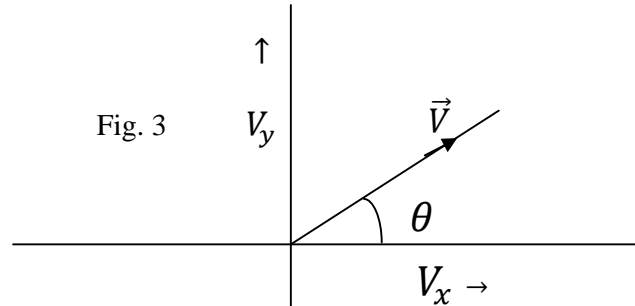
where  $V$  = magnitude of the vector and  $\theta$  = angle the direction of the vector makes with the positive x-axis.

Y-component of vector  $\vec{V}$ ,

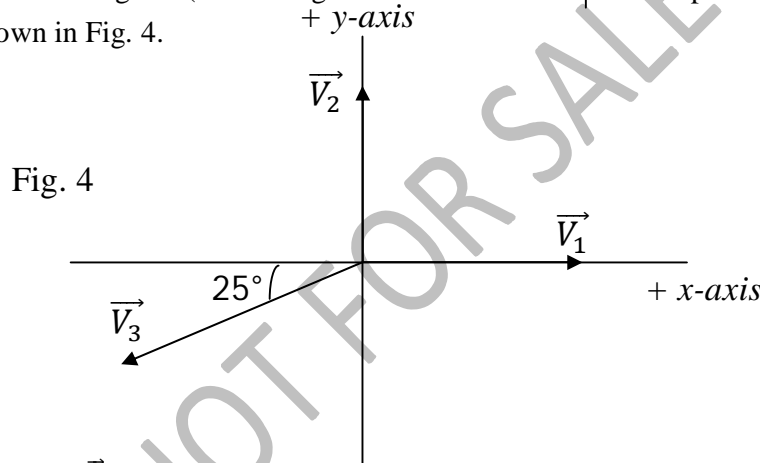
$$V_y = V\sin\theta$$

X-component of vector  $\vec{V}$ ,

$$V_x = V\cos\theta$$



**Example 1:** Determine angle  $\theta$  (i.e. the angle each vector makes with the positive x-axis) for vectors  $\vec{V}_1$ ,  $\vec{V}_2$  and  $\vec{V}_3$  shown in Fig. 4.



**Solution:**

The direction of vector  $\vec{V}_1$ , as indicated in the diagram, is along the positive x-axis

Therefore, the angle  $\vec{V}_1$  makes with the positive x-axis  $\theta_1 = 0^\circ$ .

The direction of vector  $\vec{V}_2$ , as indicated in the diagram, is along the positive y-axis, and the angle between the positive y-axis and positive x-axis is  $90^\circ$ .

Therefore, the angle  $\vec{V}_2$  makes with the positive x-axis  $\theta_2 = 90^\circ$ .

For vector  $\vec{V}_3$ , the angle  $\theta_3$  that  $\vec{V}_3$  makes with positive x-axis is  $180^\circ + 25^\circ$ ,

therefore  $\theta_3 = 205^\circ$ .

**Example 2:** Using fig. 4, if the magnitude of vectors  $\vec{V}_1$ ,  $\vec{V}_2$  and  $\vec{V}_3$  is 10 N, 15 N and 20 N respectively, calculate the y- and x-components of each of the vectors.

**Solution:**

Vector  $\vec{V}_1$ , magnitude of the vector  $V_1 = 10 \text{ N}$  and  $\theta_1 = 0^\circ$ .

x-component  $V_{1,x} = 10 \cos 0^\circ = 10 \text{ N}$ , and

y-component  $V_{1,y} = 10 \sin 0^\circ = 0 \text{ N}$

For  $\vec{V}_2$ , magnitude  $V_2 = 15 \text{ N}$ ,  $\theta_2 = 90^\circ$

$V_{2,x} = 15 \cos 90^\circ = 0 \text{ N}$  x - component

$V_{2,y} = 15 \sin 90^\circ = 15 \text{ N}$  y - component

For  $\vec{V}_3$ ,  $V_3 = 20 \text{ N}$ ,  $\theta_3 = 205^\circ$

$V_{3,x} = 20 \cos 205^\circ = 20 \times (-0.9063) = -18.13 \text{ N}$  x - component

$V_{3,y} = 20 \sin 205^\circ = 20 \times (-0.4226) = -8.45 \text{ N}$  y - component

**Note:** If the direction of a vector is along x-axis or y-axis, the vector will have only one component e.g.  $\vec{V}_2$  is along y-axis,  $V_{2,y} = 15 \text{ N}$  while  $V_{2,x} = 0 \text{ N}$ .

Now to  $\vec{F}_{\text{Net}}$  in Fig. 2, we can write each of the forces in a vector equation and the net force will be the vector sum of all the three forces.

$$\begin{aligned}\vec{F}_{\text{Net}} &= F_1 \cos 90^\circ \mathbf{i} + F_1 \sin 90^\circ \mathbf{j} \\ &\quad + F_2 \cos 30^\circ \mathbf{i} + F_2 \sin 30^\circ \mathbf{j} \\ &\quad + F_3 \cos 0^\circ \mathbf{i} + F_3 \sin 0^\circ \mathbf{j}\end{aligned}$$

$$\vec{F}_{\text{Net}} = (0 + F_2 \cos 30^\circ + F_3) \mathbf{i} + (F_1 + F_2 \sin 30^\circ + 0) \mathbf{j}$$

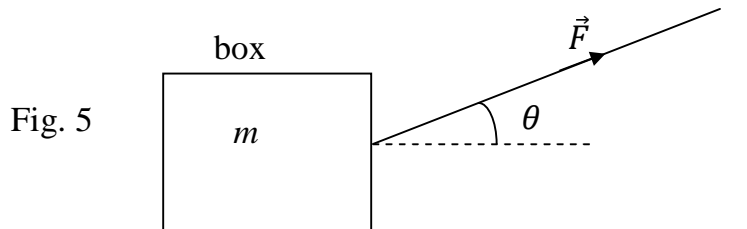
## NEWTON'S SECOND LAW OF MOTION

The net force  $\vec{F}_{\text{Net}}$  on a body is equal to the product of the body's mass (m) and its acceleration.

$$\vec{F}_{\text{Net}} = m\vec{a}$$

$\vec{F}_{\text{Net}}$  can also be written in form of its components;  $F_{\text{net},x} = ma_x$  &  $F_{\text{net},y} = ma_y$

The implication of these two equations above is that x-component of the resultant or net force is responsible for acceleration along x-axis while y-component is responsible for acceleration along y-axis. A good illustration of this is shown in Fig. 5 below.

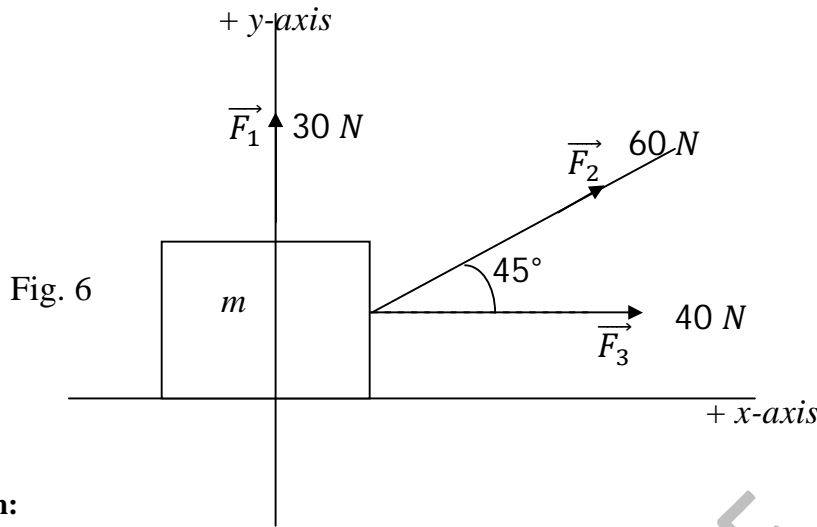


If the value of the applied or net force, in this case  $F$ , is gradually increased from zero upward, it will get to a particular value of  $\vec{F}$  at which the box will be lifted (or just about to be lifted) off the surface. At this value of  $\vec{F}$ , in the direction shown, the y-component (i.e.  $F_y$ ) is equal to the weight of the box. To keep the box moving along the horizontal surface, the applied force needs to be reduced.

Therefore, from this illustration, the x-component of the net-force ( $F_{\text{net},x}$ ) will drag the box along the horizontal surface while the y-component ( $F_{\text{net},y}$ ) will try to lift the box off the surface.

We can also state the Newton's second law of motion as: "The effect of a net force is to change the state of motion of the object on which it acts".

**Example 3:** Find the magnitude and direction of the net force acting on the object shown in Fig. 6 below.



**Solution:**

$$\vec{F}_1, F_1 = 30N, \quad \theta_1 = 90^\circ$$

$$\vec{F}_2, F_2 = 60N, \quad \theta_2 = 45^\circ$$

$$\vec{F}_3, F_3 = 40N, \quad \theta_3 = 0^\circ$$

$$\text{The net force } \vec{F}_{Net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

$$\begin{aligned}
 &= 30\cos 90^\circ i + 30\sin 90^\circ j \\
 &\quad + 60\cos 45^\circ i + 60\sin 45^\circ j \\
 &\quad + 40\cos 0^\circ i + 40\sin 0^\circ j \\
 &= (0 + 42.4 + 40.0)i + (30 + 42.4 + 0)j
 \end{aligned}$$

$$\vec{F}_{Net} = 0i + 30j$$

$$\begin{aligned}
 &+ 42.4i + 42.42j \\
 &+ 40.0i + 0j
 \end{aligned}$$

$$\vec{F}_{Net} = 82.42i + 72.42j$$

$$\text{The magnitude of } \vec{F}_{Net}, F_{Net} = \sqrt{82.42^2 + 72.42^2} = 109.72 \text{ N}$$

Angle  $\theta$  that  $\vec{F}_{Net}$  makes with positive  $x$ -axis,

$$\theta = \tan^{-1}\left(\frac{72.42}{82.42}\right) =$$

The magnitude  $F_{net} = 109.72 \text{ N}$  and  $\theta$  which indicates direction =

**Example 4:** If the mass of the object in Fig. 6 is 15Kg, find the acceleration (in magnitude and direction).

**Solution:**

$$\vec{F}_{Net} = m\vec{a}, \text{ From example 3, } \vec{F}_{Net} = 82.42i + 72.42j$$

$$\text{Therefore, } \vec{a} = \frac{F_{Net}}{m} = \frac{82.42i + 72.42j}{15}$$

$$\vec{a} = 5.495i + 4.828j$$

$$\text{Magnitude of } \vec{a}, a = \sqrt{5.495^2 + 4.828^2} = 7.31 \text{ m/s}^2$$

The angle  $\theta$ ,  $\vec{a}$  makes with positive x-axis,  $\theta = \tan^{-1} \frac{4.828}{5.495}$

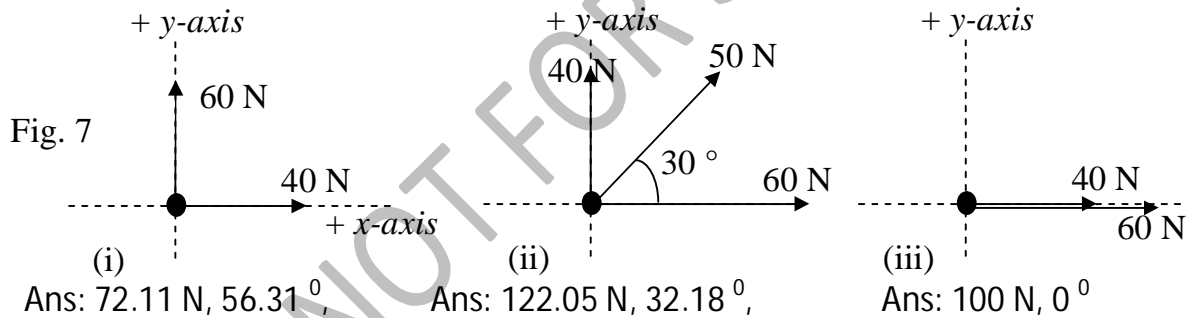
The magnitude of acceleration  $a = 7.31 \text{ m/s}^2$ , and  $\theta =$

**Alternative Solution:** From  $\vec{F}_{Net} = m\vec{a}$ , the direction of the net force  $\vec{F}_{Net}$  is the direction of the resulting acceleration. For the magnitude of acceleration, one can divide the magnitude of net force

$F_{net}$  by the mass i.e.  $a = \frac{F_{net}}{m} = \frac{109.72}{15} = 7.31 \text{ m/s}^2$

### Problems

1. A 1580Kg car is travelling at a speed of 15.0m/s. What is the magnitude of the horizontal net force that is required to bring it to a halt in a distance of 50.0m? Ans = 355N.
2. A 900Kg car travelling at 25m/s brakes hard and comes to rest in 5s. What is the average breaking force? (Ignore drag) Ans = 4500N.
3. A net force acts on mass  $m_1$  and creates an acceleration. A mass  $m_2$  is added to mass  $m_1$ . The same net force acting on the two masses together creates one-third the acceleration. Determine the ratio  $\frac{m_2}{m_1}$ . Ans = 2:1 or 2.
4. An empty plane whose mass is 30, 400Kg has a maximum takeoff acceleration of  $1.20 \text{ m/s}^2$ . What is its maximum acceleration when it is carrying a load of 8200Kg? Ignore friction. Ans =  $0.945 \text{ m/s}^2$ .
5. Find the magnitude and direction of the net force acting on each of the three objects shown in Fig. 7 below.

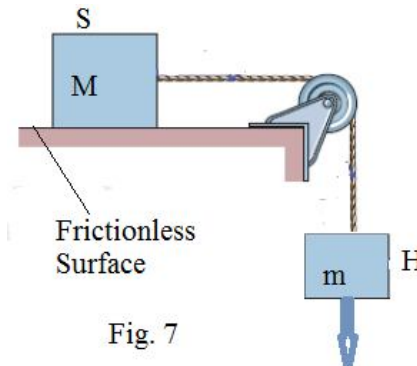


6. If the masses of the objects in Fig. 7 above are 7.5Kg, find their accelerations (both magnitude and direction). Ans:  $9.61 \text{ m/s}^2$ ,  $56.31^\circ$ ;     $16.27 \text{ m/s}^2$ ,  $32.18^\circ$ ;     $13.33 \text{ m/s}^2$ ,  $0^\circ$

### Example 4

In fig. 8 below, If  $M = 3.3\text{Kg}$  and  $m = 2.1\text{K}$  and assuming that the cord and pulley have negligible mass, find

- a. The acceleration of the sliding block
- b. The acceleration of the hanging block
- c. The tension in the cord



### Solution:

The forces and their direction acting on block S and block H are shown. Since block S is on a frictionless surface, the net force on it is T. The forces on block H are T and  $F_g$  (Please, take note of the direction of T and  $F_g$ ).

Applying second law of motion,  $\vec{F}_{Net} = m\vec{a}$

On block S,  $T = Ma$  (1)

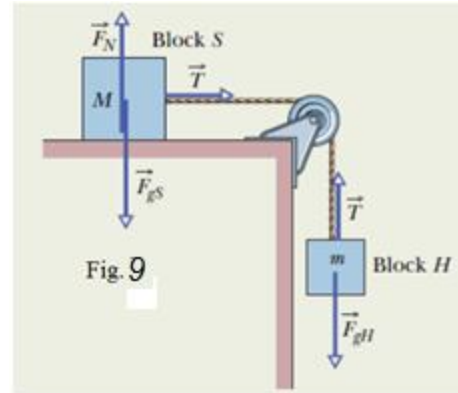
On block H,  $mg - T = ma$  (2)

Using eqn (1),  $T = Ma$ , replace T in (2) with Ma

$$Mg - Ma = ma$$

$$Mg = Ma + ma = (M + m)a$$

$$a = \frac{M}{M+m}g = \frac{2.1}{3.3+2.1} \times 10 = 3.89 \text{ m/s}^2$$



Since the cord is inextensible, both blocks (S and H) will accelerate at  $a = 3.89 \text{ m/s}^2$ .

(c) Using equation 1,  $T = 3.3 \times 3.90 = 12.83 \text{ N}$

### SECOND LAW OF MOTION (MOMENTUM)

Force is equal to the rate of change of momentum,

$F = \frac{dp}{dt}$  = force acting on a body in motion is equal to the rate of change of momentum of the body.

Where dp is change in momentum and t, the time for the change. Suppose an external force F acts on an object of mass m moving with initial velocity  $v_1$  to change its velocity to  $v_2$  at time interval t.

$$\text{Then } F = \frac{dp}{dt} = \frac{mv_2 - mv_1}{t} = \frac{m(v_2 - v_1)}{t}$$

$$\text{But } F = ma = \frac{m(v_2 - v_1)}{t},$$

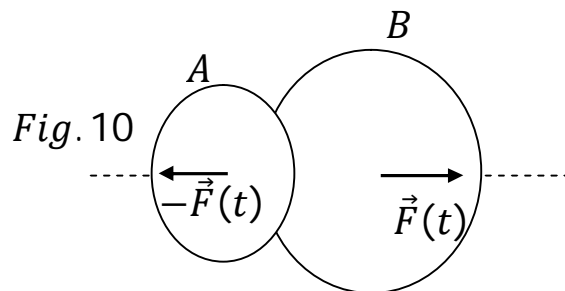
$$a = \frac{(v_2 - v_1)}{t} \text{ - definition of acceleration.}$$

This is another statement of Newton's second law of motion

### COLLISION

Collision is an isolated event in which two or more bodies exert relatively strong forces on each other for a relatively short time.

### IMPULSE AND LINEAR MOMENTUM



The diagram in Fig. 10 above shows the collision between two objects A and B of different masses. If the collision is head-on, the direction of their final velocity will be along x-axis. The two forces  $\vec{F}(t)$

and  $-\overrightarrow{F(t)}$  will change the linear momentum of both bodies. The amount of change  $dp$  is

$d\vec{p} = \overrightarrow{F(t)}dt$  according to Newton's second law of motion

Integrating over the interval  $\Delta t$  – from initial time  $t_i$  to final time  $t_f$

$$\int_{t_i}^{t_f} d\vec{p} = \overrightarrow{F(t)}dt$$

$\int_{t_i}^{t_f} d\vec{p} = \vec{p}_f - \vec{p}_i$ , the change in linear momentum of the body.

$\int_{t_i}^{t_f} \overrightarrow{F(t)}dt = \vec{J}$  i.e. the impulse

For each body, change in the linear momentum is equal to the impulse that acts on the body.

## MOMENTUM AND KINETIC ENERGY IN COLLISIONS

**Elastic Collision:** Both momentum and Kinetic energy are conserved i.e. change in momentum  $dp =$  constant and  $k_i = k_f$ .

**Inelastic Collision:** Momentum is conserved by Kinetic energy decreases.

i.e.  $dp =$  constant but  $k_i \neq k_f$ .

## LAW OF CONSERVATION OF LINEAR MOMENTUM

In a closed, isolated system containing a collision, the linear momentum of each colliding body may change but the total linear momentum  $\vec{p}$  of the system cannot change, whether the collision is elastic or inelastic.

**Collision in one dimension:** For objects moving along x-axis or y-axis, velocity has a single component. Under **in-elastic collision**, the two bodies join together after collision and move with common velocity  $v$ . Therefore, we can write;

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \quad \text{i.e. } dp = \text{constant}$$

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \quad k_i = k_f$$

**Elastic collision in one dimension:**

The two bodies of masses  $m_1, m_2$  and moving with initial velocities  $u_1, u_2$  respectively, if after collision the velocities change to  $V_1, V_2$  respectively, then we can write

$$m_1u_1 + m_2u_2 = m_1V_1 + m_2V_2 \quad \text{i.e. } dP = \text{Constant}$$

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2 \quad \text{i.e. } k_i = k_f$$

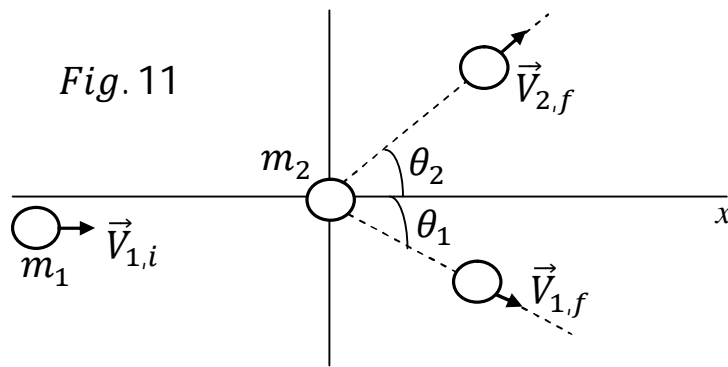
## Collision in Two Dimensions

When collision between two bodies is not head-on, the direction of their final velocities is not along their initial axis. For this type of collision, linear momentum is still conserved.

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$

If collision is elastic, total kinetic energy is conserved.

$$k_{1i} + k_{2i} = k_{1f} + k_{2f}$$



From Fig. 11 above, after collision, the impulse between the bodies sends the bodies off at angles  $\theta_1$  and  $\theta_2$  to the  $x$  axis.

Momentum is a vector quantity, so the components along  $x$ -axis is

$$m_1 V_{1i} = m_1 V_{1f} \cos \theta_1 + m_2 V_{2f} \cos \theta_2$$

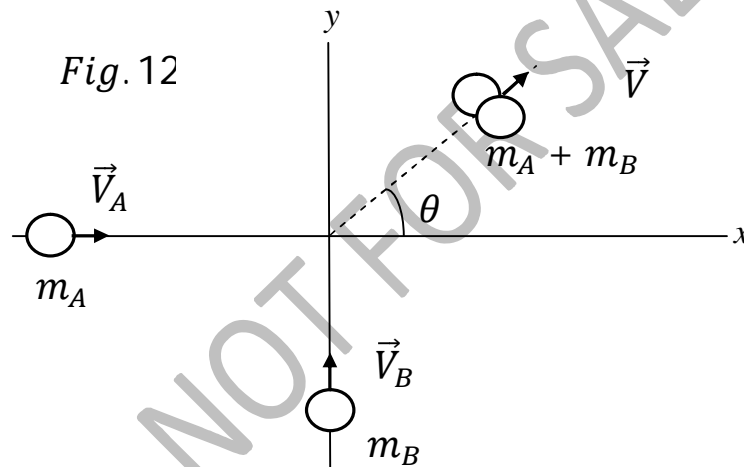
And along  $y$ -axis is

$$0 = -m_1 V_{1f} \sin \theta_1 + m_2 V_{2f} \sin \theta_2$$

Also, kinetic energy is conserved.

$$\frac{1}{2} m_1 V_{1i}^2 = \frac{1}{2} m_1 V_{1f}^2 + \frac{1}{2} m_2 V_{2f}^2$$

Example



Two objects A and B collide and embrace, in a completely inelastic collision. Thus, they stick together after impact as shown in the diagram above, where the origin is placed at the point of collision. A is of mass 83 Kg originally moving east with speed 6.2 Km/h and B is of mass 55 Kg and is originally moving north with speed 7.8 Km/h. What is the common velocity  $\vec{v}$ .

Questions

1. A 1600Kg van moving at  $20\text{ms}^{-1}$  crashes into the back of a small car of mass 700Kg traveling in the same direction at  $15\text{ms}^{-1}$ .
  - a) What is the velocity of the two vehicles immediately after the collision of the lock together?
  - b) How much kinetic energy is transferred to other forms in the collision?
 Ans: (a) 18.478 m/s (b) 6098.04 J
2. In an experiment using a linear air track a, 0.5Kg rider travelling to the right at  $2\text{ms}^{-1}$  collides with a stationary rider of mass 1Kg. Find the velocities of the two riders after the collision if:
  - a. It is perfectly elastic Ans:  $V_1 = -0.667 \text{ m/s}$   $V_2 = 1.33 \text{ m/s}$
  - b. It is totally inelastic. Ans: 0.667 m/s

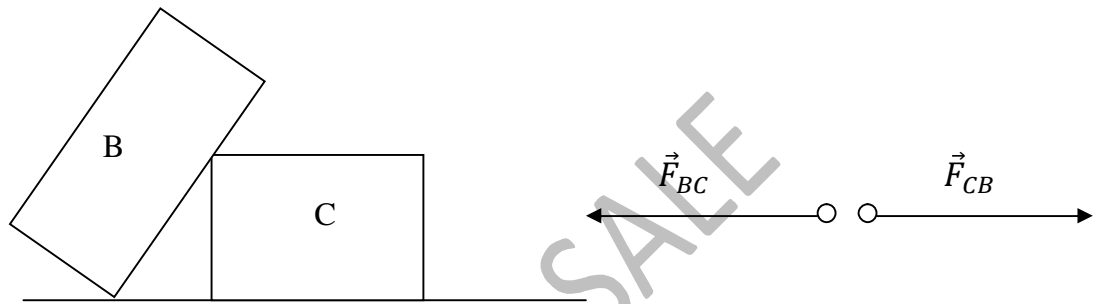


3. The National Transportation Safety Board is testing the crash-worthiness of a new car. The 2300Kg vehicle moving at 15 m/s is allowed to collide with a bridge abutment, which stops it in 0.56 s. What is the magnitude of the average force that acts on the car during the impact?  
Ans: 61607.1 N
4. A 150 g baseball pitched at a speed of 40 m/s is hit straight back to the pitcher at a speed of 60 m/s. What is the magnitude of the average force on the ball if the bat is in contact with the ball for 5.0 ms? Ans: 3,000 N
5. Two 2.0Kg bodies, A and B collide. The velocities before the collision are  $\vec{V}_A = 15i + 30j$  and  $\vec{V}_B = -10i + 5.0j$ . After the collision,  $\vec{V}_A = -5.0j + 20j$ . All speeds are given in meters per second. (a) What is the final velocity B? (b) How much Kinetic energy is gained or lost in the collision? Ans: (a)  $10i + 15j$  (b) 525 J

### NEWTON'S THIRD LAW OF MOTION

When two objects interact, the forces on the objects from each other are always equal in magnitude and opposite in direction.

Fig. 13



Consider the two objects shown in Fig. 13 box B is leaning on crate C, the third law can be written for this arrangement as a scalar relation.

$$F_{BC} = F_{CB} \quad \text{i.e. equal magnitude}$$

As a vector relation,

$$\vec{F}_{BC} = -\vec{F}_{CB} \quad \text{i.e. equal magnitude and opposite direction}$$

Where  $F_{BC}$  is the force on the box from the crate

$F_{CB}$  is the force on the crate from the box.

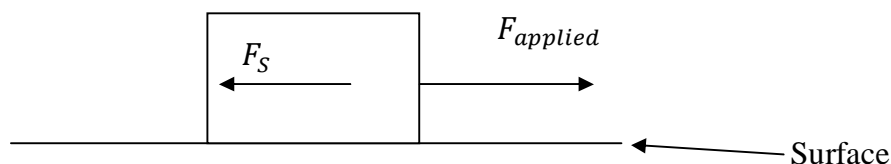
### FRICTION

Friction is the force exerted by a surface as an object moves across it or makes an effort to move across it. From this definition, friction is called to play only when an object moves or tries to move across a surface.

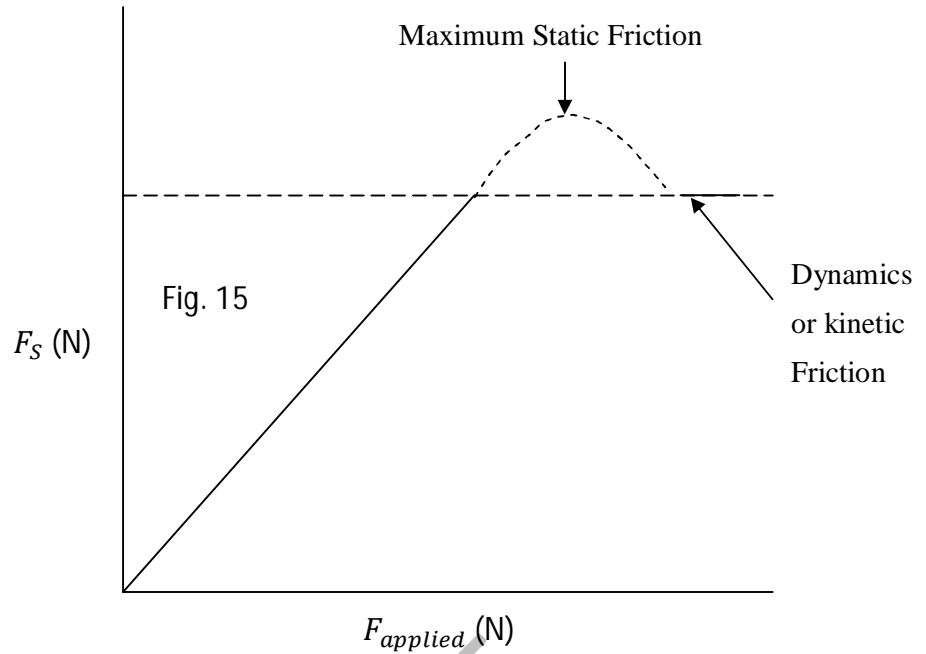
Consider an object placed on a surface as shown in fig. 13. If there is no movement of the object, though there is an applied force  $F_{\text{applied}}$ , in the direction shown, then

$$\text{Frictional force } F_s = F_{\text{applied}} \quad (\text{i.e. if } a=0)$$

Fig. 14



The implication of the above equation is that  $F_s$  is equal to the applied force ( $F_{\text{applied}}$ ) as long as there is no movement of the object. Fig. 15 is an illustration of the relationship between the applied force and static friction  $F_s$ . The relation is broken just as the object is about to move. At this point, we have what is called maximum static friction  $F_{s,\text{max}} = \mu_s F_N$ , where  $\mu_s$  is called coefficient of static friction and  $F_N = mg$  is the normal reaction or weight of the object.



The moment the object starts to move, dynamic friction or kinetic friction is called into play.

Dynamic friction  $F_k = \mu_k F_N$ , where  $\mu_k$  is the coefficient of dynamic friction.

Static frictional force acts between surfaces at rest when a force is applied to make them slide over each other. Kinetic frictional force acts between surfaces as they slide over each other.

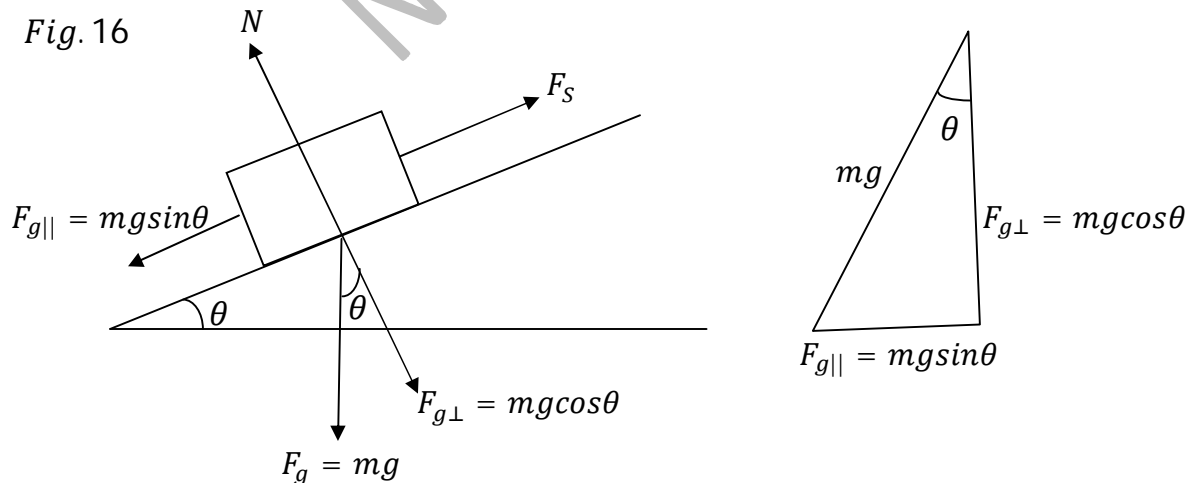
Factors upon which friction depends

Friction depends on:

1. The nature of the two surfaces in contact
2. In the case of an object placed on a surface, friction depends on the weight of the object.

Note: Friction does not depend on the amount of surface area in contact or the relative speed of the bodies.

### Inclined Plane



Inclined plane can as well be treated like horizontal surface in which forces act along the surface. Fig. 16 shows an object placed on an inclined plane and the forces. In this case, not only forces parallel to the surface and their directions are to be considered.

To be considered are the forces acting along the surface of the inclined plane, parallel component of the weight of the object  $F_{g\parallel} = mg \sin \theta$  and the friction  $F_s = \mu N$  ( $N$ =Normal Reaction). The right

angle triangle on the right of the figure shows the weight and the parallel ( $F_{g11}$ ) and vertical ( $F_{g\perp}$ ) components of the weight.

If no external force acts on the object, the object will start to slide when  $F_{g11}$  is equal to the maximum static friction (i.e.  $F_{s,max}$ ). At that point, one can estimate the coefficient of static friction of the inclined plane.

$$F_{s,max} = F_{g11} = mgsin\theta$$

But  $F_{s,max} = \mu_s N$  and  $N = F_{g\perp} = mgcos\theta$  (i.e. from 3<sup>rd</sup> law)

$$\mu_s mgcos\theta = mgsin\theta$$

$$\mu_s = \frac{mgsin\theta}{mgcos\theta} = \frac{sin\theta}{cos\theta} = tan\theta$$

$$\mu_s = tan\theta$$

## WORK AND ENERGY

Work is done when an object is moved through a distance in the direction of the applied force, while energy is the ability to do work. The unit of work and that of energy is in Joule (J).

Work done by a constant force

Suppose an object moves from an initial point  $x_i$  to a final point  $x_f$  so that the displacement of the point the force acts on is positive i.e.

$$\Delta x = x_f - x_i > 0$$

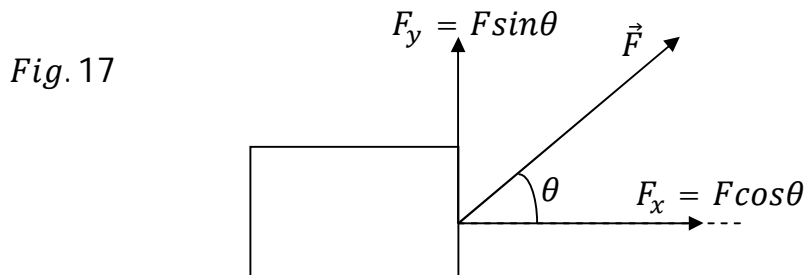
Then, workdone  $W = F_{\text{applied}} \cdot \Delta x$

Work is a scalar quantity and the unit is in (J).

Work done against friction,  $W_{\text{friction}} = -\mu_K mg\Delta x$

### Work done by a constant force applied at an angle

If the direction of the applied force is at an angle  $\theta$  to the horizontal, then the x-component of the applied force is responsible for the motion of the object along the horizontal.



Work done  $W = F_x \times \Delta x$

**Conservation of Energy:** The law of conservation of energy states that the total energy of a closed system is constant.

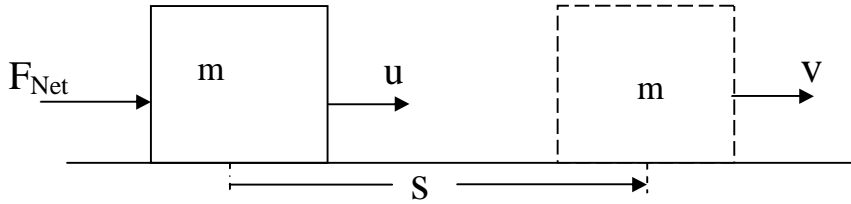
## WORK AND KINETIC ENERGY

From the definition of work, applied force must have moved an object through a distance – meaning that it causes the object's speed to change. Hence, the work done on an object can be expressed as a function of the object's final speed  $v$  and mass  $m$  i.e. Kinetic energy of the object.

$$K.E. = \frac{1}{2}mv^2$$

## WORK-ENERGY THEOREM

Fig. 18



Consider the diagram in Fig. 18 above, assuming a net force  $F_{Net}$  in the direction indicated causes an object of mass  $m$  to move through a distance  $S$ . Then from the Newton's 2<sup>nd</sup> law of motion,

$$\vec{F}_{Net} = m\vec{a},$$

For a motion under constant acceleration, we write

$$2ax = v^2 - u^2 \text{ as } \frac{v^2 - u^2}{2}$$

Where  $a$  is the acceleration and  $x$  or  $s$  is the distance or displacement.

From the definition of work,

$$W_{net} = F_{net} \cdot S = mas = m \frac{v^2 - u^2}{2}$$

$$W_{net} = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = KE_f - KE_i = \Delta KE$$

We can therefore say that total net work done is equal to the change in Kinetic energy.