

## PHY 115 LECTURE NOTE

(S. Olatunji)

(CONTENT: Young Modulus, Fluid Flow, Turbulence, Streamline)

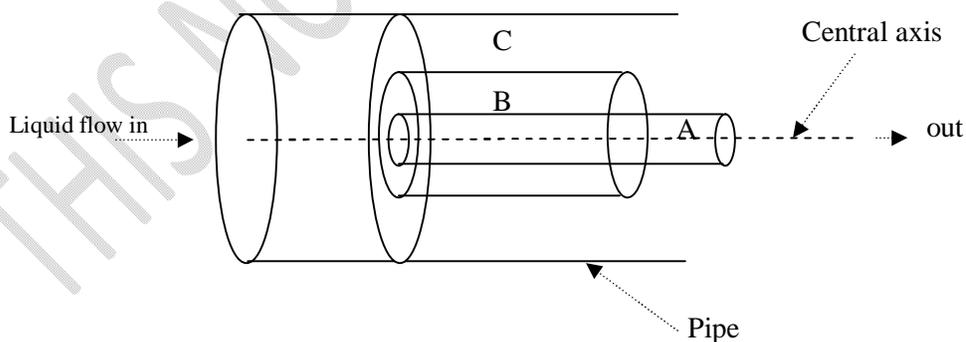
### FLUID FLOW

Fluid refers to liquid and gas. Hydrostatics is the study of fluid at rest and hydrodynamics is the study of fluid in motion. In studying fluid flow, parameters such as velocity, pressure, density and temperature are considered. When parameters such as electric current and magnetism are studied in addition, such endeavour is termed magnetohydrodynamics. If we assume that density and pressure determine the temperature in a magnetohydrodynamics fluid then,  $PV = nRT$  and density  $\rho = nM/V$ ; where  $M$  is molar mass. An incompressible fluid is one with constant density.

Some characteristics of fluid flow include: steady/laminar flow, turbulent/non steady flow, viscous flow, non viscous flow, rotational and non rotational flows.

### LAMINAR/STEADY AND TURBULENT FLOWS

Consider a moving liquid in a stationary pipe



Let the liquid be moving in layers A, B and C. Liquid in layer A moves faster than B and B is faster than C. This is due to high frictional force effects at the pipe's edges (caused by adhesion between the particles of the liquid and pipe), which is stationary around the moving liquid layers.

The flow of liquid in which the layers are distinct and the layers are not crossing such that there is frictional force within the layers is called laminar flow. In a laminar flow, velocity of particles at a given equidistance from the central axis of motion is constant. Further, if pressure, density and velocity of a moving fluid are independent of time then flow is laminar /steady. This is achievable at low flow speeds.

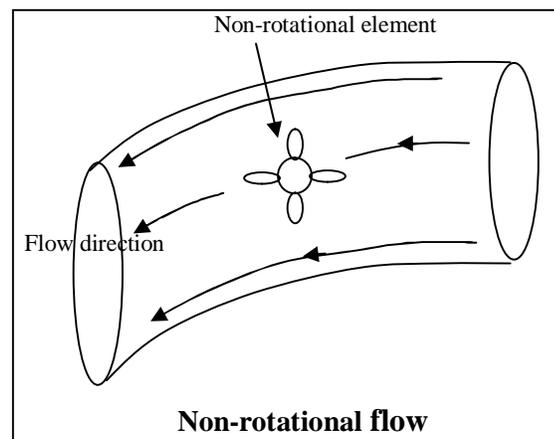
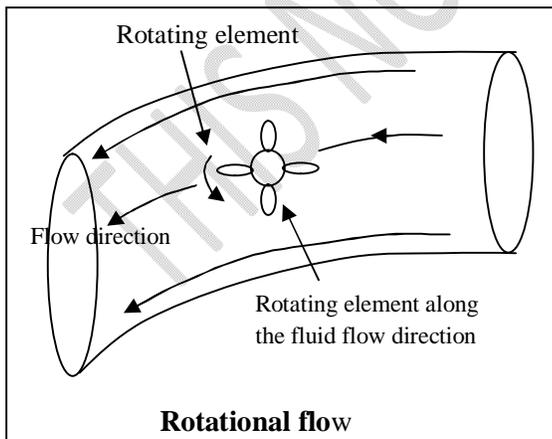
If the layers of a flowing fluid intermix, it is a turbulent flow. In this case, velocity could be a complicated function of time e.g. Tides, water-falls etc.

## VISCOUS AND NON VISCOUS FLOWS

Viscosity in fluid is analogous to friction in solid motion. A flow of fluid in which no energy is dissipated through viscous forces is called a non viscous flow; otherwise, it is called a viscous flow.

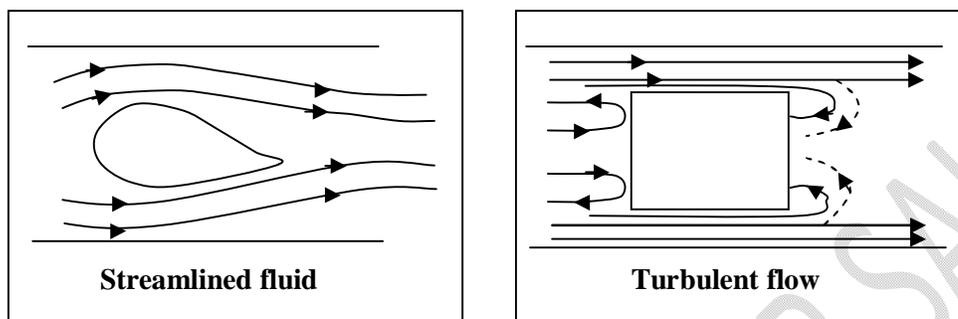
## ROTATIONAL AND NON ROTATIONAL FLOWS

If an element of the fluid does not rotate about an axis through the centre of mass of the element, the flow is said to be non rotational, otherwise, it is rotational.



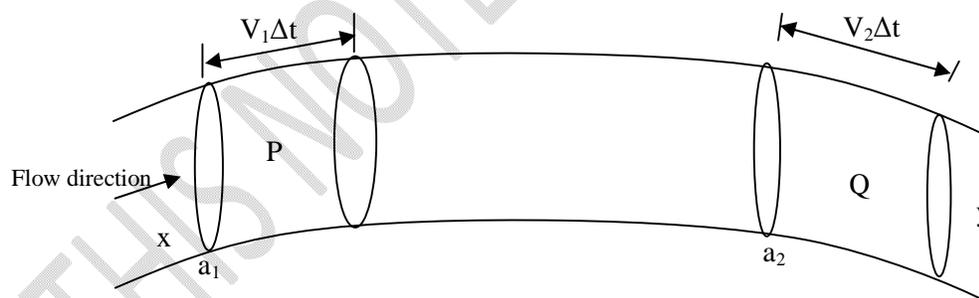
## STREAMLINES IN A MOVING FLUID

Streamline is a curve whose tangent always lies along the direction of motion/velocity of the fluid at that point.



Line of flow is the path followed by an element of a moving fluid. In the steady/laminar flow, the streamlines coincide with the line of flow. No two streamlines can cross each other, this is because if they did, and oncoming fluid particle could go either one way or the other, and hence the flow could not be steady.

## VOLUME FLOW RATE



Suppose a fluid flows in at  $x$ , and moves out at  $y$ . Let  $a_1$  and  $a_2$  be the cross-sectional areas of the pipe at  $x$  and  $y$ . Let  $V_1$  and  $V_2$  be the speeds within sections  $P$  and  $Q$  after the passage of a very small time  $\Delta t$ .

Volume across  $a_1$  within time  $\Delta t = a_1 V_1 \Delta t$ ,

and volume  $a_2$  within the same  $= a_2 V_2 \Delta t$

If  $\rho_1$  and  $\rho_2$  are the densities at P and Q then the mass of fluid crossing  $a_1$  (i.e.  $\Delta m_1$ )  $= \rho_1 a_1 v_1 \Delta t$

This is equal to that around  $a_2$   $a_1$  (i.e.  $\Delta m_2$ )  $= \rho_2 a_2 v_2 \Delta t$ .

By conservation of mass rule  $\Delta m_1 = \Delta m_2$

That is  $\rho_1 a_1 v_1 \Delta t = \rho_2 a_2 v_2 \Delta t = \dots\dots\dots$

In general,  $\rho a v = \text{constant}$ ----- (1)  $\leftarrow$  flow continuity equation.

For an incompressible fluid, density is constant thus (1) becomes

$$a v = \text{constant} = R$$

i.e.,  $a v = R$  ----- (2)

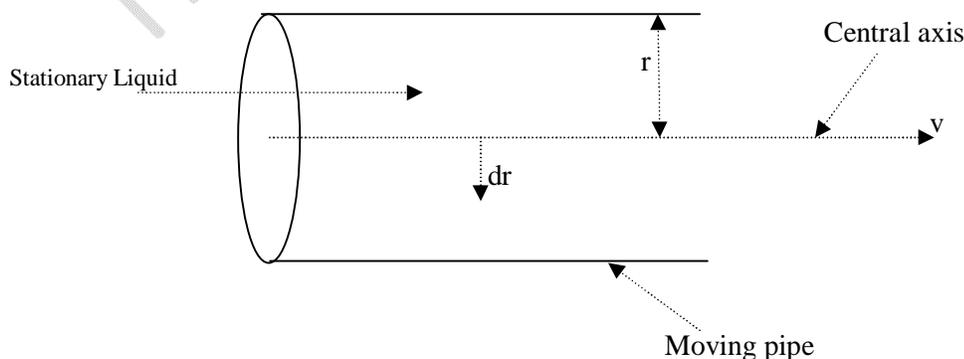
ie,  $R = \text{volume flow rate}$  (ie. volume flux)

Equation (2) implies that in an incompressible fluid flow the speed of flow is inversely proportional to the cross sectional area.

$$\Rightarrow v = R (1/a) \text{ ----- (3)}$$

## POISEUILLE'S EQUATION

Consider a stationary liquid in a moving pipe. The velocity will be much, near the edges of the pipe and decrease towards the centre as the liquid drifts along the pipe's direction. This is a reverse of a moving liquid in a stationary pipe.



At a small distance  $dr$  away from the central axis, there is a small change in the velocity  $dv$ .

Thus, velocity gradient =  $dv/dr$

Viscosity is found to be proportional to the velocity gradient and cross-sectional area of the pipe

ie  $F \propto A \frac{dv}{dr}$

ie  $F = \eta A \cdot \frac{dv}{dr}$ , where  $\eta$  = coefficient of viscosity

ie,  $\eta = \frac{F/A}{dv/dr} = \frac{\text{tangential stress}}{\text{velocity gradient}}$

its SI unit = Pas

Also, for fluid flowing in a pipe, volume per second (flow rate)  $V$  depends on radius of the pipe; pressure gradient ( $\Delta P/L$ ) and on  $\eta$

i.e.  $V = \frac{V}{t} = k\eta r \frac{\Delta P}{L}$  ----- (4)

By dimension method,  $\frac{V}{t} = k\eta^x r^y \left(\frac{\Delta P}{L}\right)^z$  ----- (5)

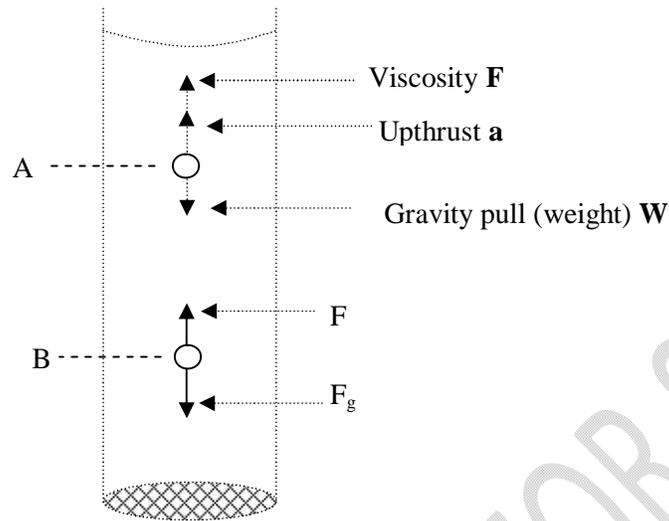
If  $k$  (constant of the proportionality) =  $\pi/8$  and solving for the values of  $x$ ,  $y$  and  $z$ , we have

$V = \frac{\pi r^4 \Delta P}{8\eta L}$   $\longleftarrow$  Poiseuille's equation

**Exercise1:** Prove (5) from (4) above.

**Exercise 2:** A 1km length pipe is to convey oil with coefficient of viscosity 0.3 Pas up to a tank. Calculate the pump pressure needed to maintain a flow rate of  $0.05\text{m}^3/\text{s}$  if the diameter of the pipe is 15 cm.

## STOKE'S LAW



Consider a sphere falling through a viscous fluid, Its velocity increases due to gravity pull until a constant velocity called **TERMINAL VELOCITY**  $V_t$  is attained. At this velocity, viscosity is balanced by the gravitational force.

Viscosity force  $F$  depends on terminal velocity, radius of the sphere and coefficient of viscosity  $\eta$

Hence  $F = KV_t r \eta$

By dimension method,

$$F = KV_t^x r^y \eta^z \text{ ----- (6)}$$

And if  $k = 6\pi$ , then

$$F = 6\pi\eta V_t \text{ ----- (7)} \quad \longleftarrow \text{ stoke's law (equation)}$$

**Exercise 3:** Prove (7) from (6)

The resultant downward force  $F$  at A is  $F = W - U - F$  which causes the sphere to accelerate to terminal velocity. If  $\rho$  is the density of the fluid and  $\delta$  is that of the sphere then

$$w = \frac{4}{3} \pi r^3 \delta g, \text{ and}$$

$$w = \frac{4}{3} \pi r^3 \rho g; \text{ So}$$

from  $F = W - U$

$$F = \frac{4}{3} \pi r^3 g (\delta - \rho) \text{ ----- (8)}$$

Equate (7) and (8)

$$\text{We have } V_t = \frac{2gr^2(\delta-\rho)}{9\eta}$$

### YOUNG'S MODULUS (Y)

Y is defined as the relation:  $\frac{\text{tensile stress}}{\text{tensile strain}}$

$$\text{Stress} = \frac{F}{A} = \frac{\text{Applied force}}{\text{Cross sectional Area}}$$

$$\text{Strain} = \frac{e}{L} = \frac{\text{Extension}}{\text{Original Length}}$$

Graph of stress/Strain gives Y by its slope.

**Exercise 4:** Sketch the graph to obtain Young's Modulus Y.

**Exercise 5:** Incompressible liquid flows in a horizontal tube AB from A to B. If the areas of cross-sections are  $30\text{cm}^2$  and  $15\text{cm}^2$  and velocity at end A is  $12\text{m/s}$ , calculate the velocity at the end B.

**Exercise 6:** YOU ARE ADVISED TO ATTEMPT MORE RELATED QUESTIONS FROM SUITABLE TEXT BOOKS ON THE TOPICS TREATED; I.E. DO NOT LIMIT YOUR STUDY EFFORTS TO THIS NOTE.