



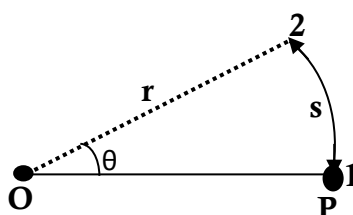
PHY 115: MECHANICS & PROPERTIES OF MATTER I

1.0 DYNAMICS OF A RIGID BODY

The study of motion which describes the way rigid objects rotate to determine their velocity, their acceleration, and their other parameters. Dynamics consider the forces that affect the motion of rotating objects. A **rigid** object is one that is non-deformable, that is, the relative locations of all particles of which the object is composed remain constant under the influence of applied force.

1.1 ROTATION OF A RIGID BODY

Suppose a point P (figure below) moves along a circular path from 1 to 2, in time Δt (at point 1, $t = 0$),



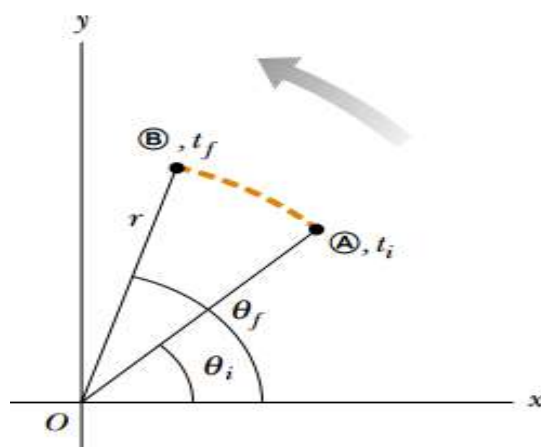
r is the radius of the circle and θ is the angle line OP moved through, from point 1 to 2, s is the length of the circular arc travelled by P.

$$\text{Generally, } \theta = \frac{s}{r} (\text{radians}) \quad 1.1$$

Because θ is the ratio of an arc length and the radius of the circle, it is a pure number. However, we commonly give θ the artificial unit radian (rad), where one radian is the angle subtended by an arc length equal to the radius of the arc, i.e.,

$$1 \text{ revolution} = 360^\circ = 2\pi \text{ rad}; 1 \text{ rad} = \frac{360^\circ}{2\pi} = 57.3^\circ = 0.159 \text{ rev.}$$

Figure of a particle on a rotating rigid object moves from A to B along the arc of a circle. In the time interval $\Delta t = t_f - t_i$, the radius vector moves through an angular displacement $\Delta\theta = \theta_f - \theta_i$:





Angular displacement ($\Delta\theta$): For a particle on a rotating rigid object such that it moves from point 1 to 2 along the arc of a circle (Figure above) from an initial angle θ_i to θ_f the angular displacement is

$$\Delta\theta = \theta_f - \theta_i \quad 1.2$$

The **average angular velocity $\bar{\omega}$** of a rotating rigid object is the ratio of the angular displacement $\Delta\theta$ to the time interval Δt .

$$\text{That is, } \bar{\omega} = \frac{\Delta\theta}{\Delta t} = \frac{\theta_f - \theta_i}{t_f - t_i} \quad 1.3$$

The unit is radians per second (rad/s).

Note : ω is positive for counterclockwise motion and negative for clockwise motion.

The **average angular acceleration $\bar{\alpha}$** of an object is defined as the ratio of the change in the angular speed to the time Δt it takes the object to undergo the change.

$$\text{That is, } \bar{\alpha} = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_i}{t_f - t_i} \quad 1.4$$

The unit is radians per square second (rad/s^2).

The quantities θ , ω , and α characterize the rotational motion of the entire rigid object as well as individual particles in the object. Using these quantities, we can greatly simplify the analysis of rigid-object rotation.

Angular position (θ), angular speed (ω), and angular acceleration (α) are analogous to linear position (x), linear speed (v), and linear acceleration (a). The variables θ , ω , and α differ dimensionally from the variables x , v , and a only by a factor having the unit of length.

The relationship between linear velocity and the angular velocity of a particle, moving along a circular path, at a point is $v = \omega r$. 1.5a

The relationship between the linear acceleration and the angular acceleration is given by:

$$a = r\alpha \quad 1.5b$$



Rotational Motion About Fixed Axis

Linear Motion

$$\begin{aligned}\omega_f &= \omega_i + \alpha t \\ \theta_f &= \theta_i + \omega_i t + \frac{1}{2} \alpha t^2 \\ \omega_f^2 &= \omega_i^2 + 2\alpha(\theta_f - \theta_i) \\ \theta_f &= \theta_i + \frac{1}{2}(\omega_i + \omega_f) t\end{aligned}$$

$$\begin{aligned}v_f &= v_i + at \\ x_f &= x_i + v_i t + \frac{1}{2} at^2 \\ v_f^2 &= v_i^2 + 2a(x_f - x_i) \\ x_f &= x_i + \frac{1}{2}(v_i + v_f) t\end{aligned}$$

1.1.1 ROTATION OF A RIGID BODY WITH UNIFORM ANGULAR ACCELERATION

The **tangential component of the linear acceleration** of a point on a rotating rigid object equals the point's distance from the axis of rotation multiplied by the angular acceleration.

$$a_t = r\alpha \quad 1.6a$$

The radial acceleration: we can also express the radial acceleration at that point in terms of angular speed as

$$a_r = \frac{v^2}{r} = \omega^2 r \quad 1.6b$$

The total linear acceleration vector at the point is $\mathbf{a} = \mathbf{a}_t + \mathbf{a}_r$, where the magnitude of \mathbf{a}_r is also the centripetal acceleration \mathbf{a}_c . Because \mathbf{a} is a vector having a radial and a tangential component, the magnitude of \mathbf{a} at the point P on the rotating rigid object is

$$a = \sqrt{a_t^2 + a_r^2} = r\sqrt{\alpha^2 + \omega^4} \quad 1.6c$$

Example 2: A wheel rotates with a constant angular acceleration of 3.50 rad/s^2 .

(a) If the angular speed of the wheel is 2.00 rad/s at $t_i = 0$, (i) through what angular displacement does the wheel rotate in 2.00 s ?

Solution:

$$\begin{aligned}\Delta\theta &= \theta_f - \theta_i = \omega_i t + \frac{1}{2} \alpha t^2 \\ &= (2 \text{ rad/s})(2 \text{ s}) + \frac{1}{2} (3.5 \text{ rad/s}^2)(2 \text{ s})^2 \\ &= 11.0 \text{ rad}\end{aligned}$$

(ii) Convert 11 radians to degrees using the equation in section 10.5

Solution:

$$\text{If } 1 \text{ rad} = 57.3^\circ, \text{ then } 11.0 \text{ rad will be } = 11.0 \times 57.3^\circ = 630.3^\circ$$

(b) Through how many revolutions has the wheel turned during this time interval?



Solution: We multiply the angular displacement found in part a(i) by a conversion factor to find the number of revolutions: (note that 1 rev = 360°)

$$\Delta\theta = 630.3^\circ \times \left(\frac{1 \text{ rev}}{360^\circ}\right) = 1.75 \text{ rev}$$

(c) What is the angular speed of the wheel at $t = 2.00 \text{ s}$?

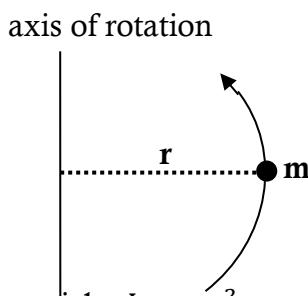
Solution:

$$\begin{aligned}\omega_f &= \omega_i + \alpha t = 2 \text{ rad/s} + (3.50 \text{ rad/s}^2)(2 \text{ s}) \\ &= 9.00 \text{ rad/s}\end{aligned}$$

1.2 MOMENT OF INERTIA OF A RIGID BODY

The resistance of an object to change its state of rotational motion about an axis is known as moment of inertia (or rotational inertia). It depends on the mass of the object and the distribution of its mass from the axis of rotation.

Consider a particle of mass m rotating about an axis through a radius r :



The moment of inertia of the particle, $I = mr^2$ 1.7a

However, if we have a rigid body that consists of n particles of masses $m_1, m_2, m_3, \dots, m_n$ at distances $r_1, r_2, r_3, \dots, r_n$ from the axis of rotation,

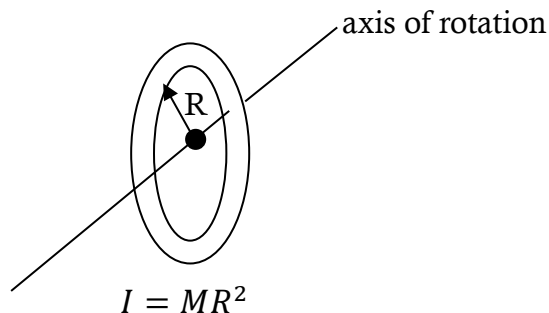
moment of inertia, $I = (m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \dots + m_nr_n^2)$

$$= \sum_{i=1}^n m_i r_i^2 \quad 1.7b$$

In integral form, $I = \int_{i=1}^n r_i^2 dm$ 1.7c

1.2.1 MOMENT OF INERTIA OF SOME BODIES

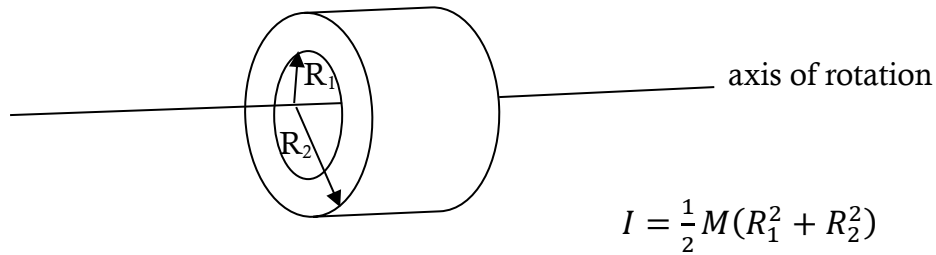
i. Hoop (Ring) about central axis



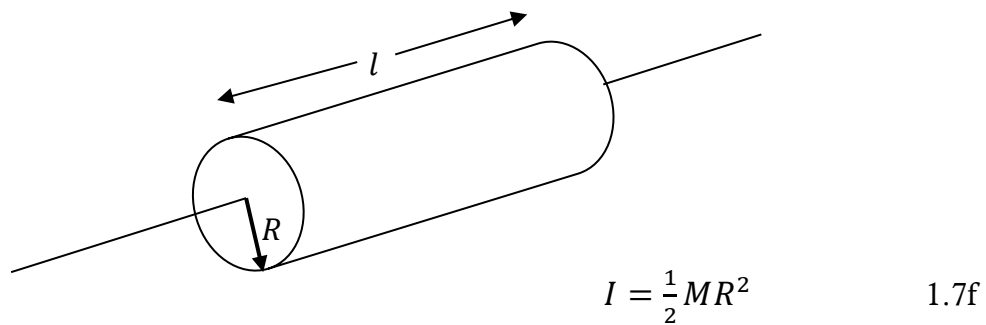
1.7d



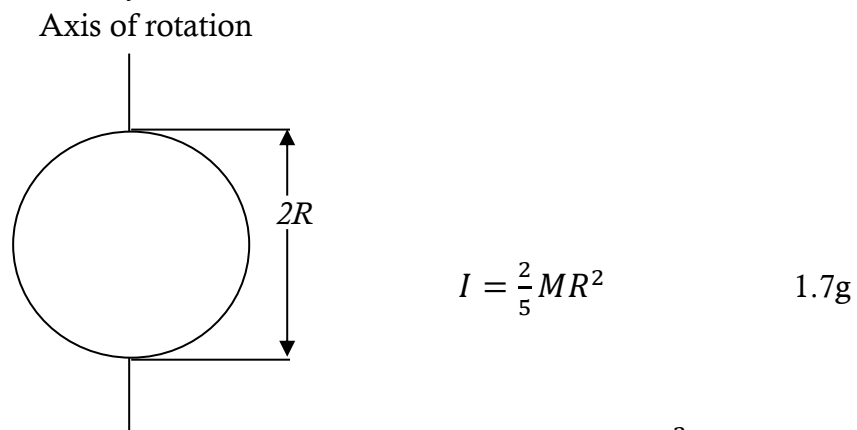
- ii. Annular cylinder about central axis



- iii. Solid cylinder (or disk) about central axis

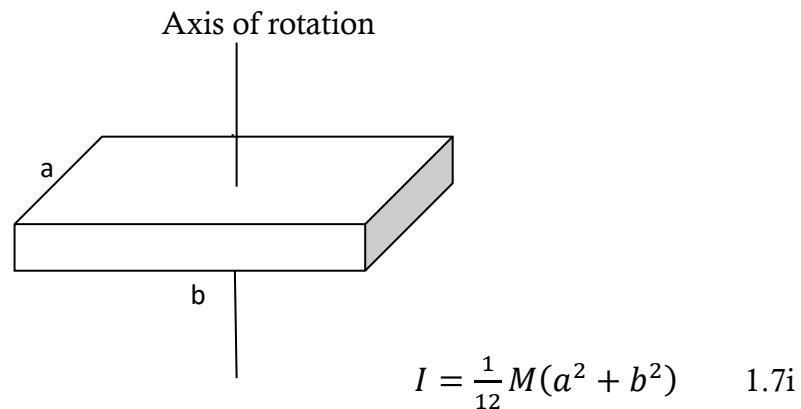


- iv. Solid sphere about any diametrical axis



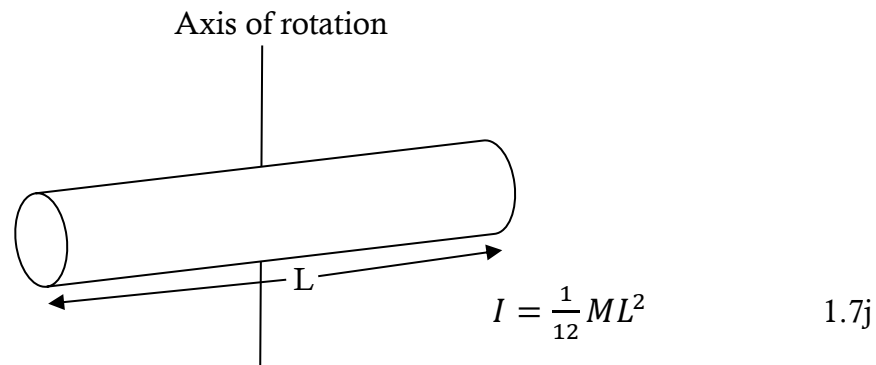
Note: For a Thin spherical shell, the moment of inertia is $I = \frac{2}{3} MR^2$ 1.7h

- v. Rectangular slab about perpendicular axis through centre





- vi. Thin rod about axis through its centre perpendicular to length

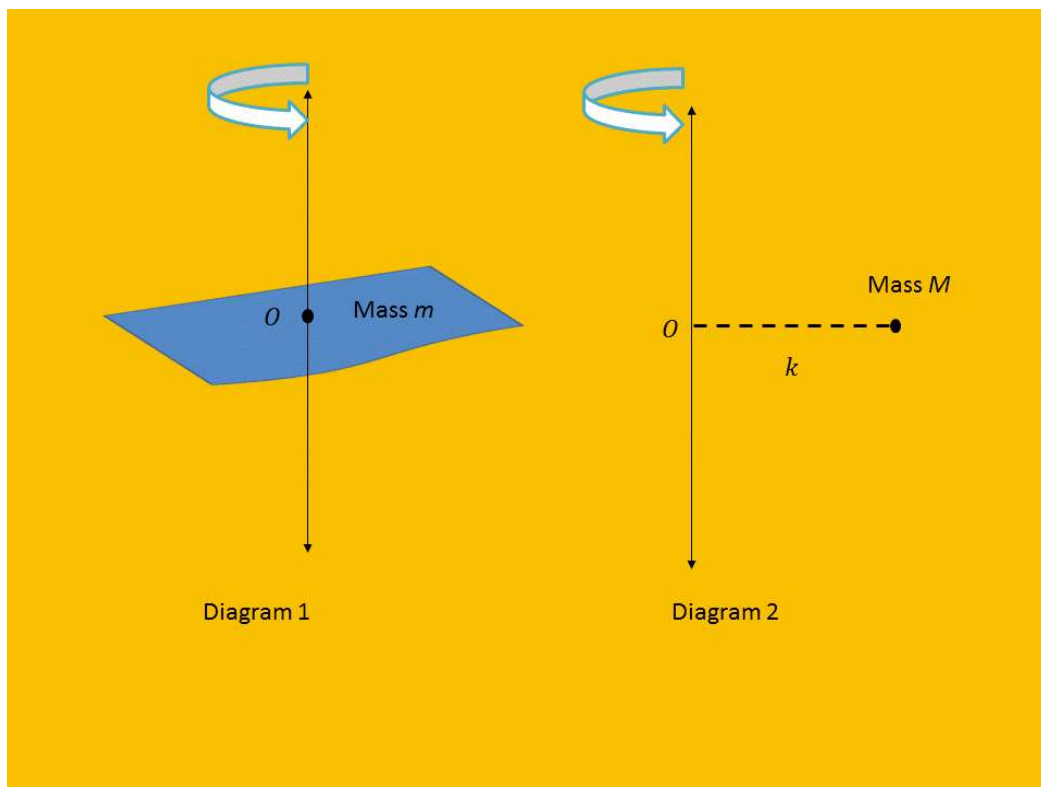


Note: The moment of inertia of the thin rod about axis through its end is

$$I = \frac{1}{3} ML^2. \quad 1.7k$$

1.3 RADIUS OF GYRATION

The moment of inertia of a body about an axis is sometimes represented using the radius of gyration. Radius of gyration is defined as the imaginary distance from the centroid at which the area of cross section is imagined to be focused at a point in order to obtain the same moment of inertia. Simply, gyration is the distribution of the components of an object.



Consider the two diagrams above.

In diagram 1, we have a shape rotating about an axis. If the mass is m then its moment of inertia is

$$I = mr^2 \quad 1.8$$



In diagram 2, we have the same axis of rotation but this time the same mass is concentrated at a point. Its moment of inertia is

$$I = Mk^2 \quad 1.9a$$

If the two values of the moments of inertia are the same, then k is called the radius of gyration

In other words, the radius of gyration is the distance from the axis of rotation to a point where the total mass of the body is supposed to be concentrated, so that the moment of inertia about the axis may remain the same. Or, the distance from the axis of rotation at which the total mass of a body is assumed concentrated and at which the moment of inertia will be equal to the actual moment of inertia of the body.

So, $I = Mk^2$

$$k = \sqrt{\frac{I}{M}} \quad 1.9b$$

Suppose a body consists of n particles each of mass m . Let $r_1, r_2, r_3, \dots, r_n$ be their perpendicular distances from the axis of rotation. Then, the moment of inertia I of the body about the axis of rotation is

$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \dots + m_n r_n^2 \quad 1.10a$$

If all the particles are of same mass m , then

$$\begin{aligned} I &= m(r_1^2 + r_2^2 + r_3^2 + \dots + r_n^2) \\ &= \frac{nm(r_1^2 + r_2^2 + r_3^2 + \dots + r_n^2)}{n} \end{aligned} \quad 1.10b$$

Since $nm = M$, total mass of the body,

$$I = \frac{M(r_1^2 + r_2^2 + r_3^2 + \dots + r_n^2)}{n} \quad 1.10c$$

From the above equations, we have

$$\begin{aligned} Mk^2 &= \frac{M(r_1^2 + r_2^2 + r_3^2 + \dots + r_n^2)}{n} \\ k &= \sqrt{\frac{(r_1^2 + r_2^2 + r_3^2 + \dots + r_n^2)}{n}} \quad 1.10d \\ &= \text{root mean square distance of the particles from the axis of rotation} \end{aligned}$$

Therefore, the radius of gyration of a body about a given axis may also be defined as the root mean square distance of the various particles of the body from the axis of rotation.

Radius of gyration can be applied to various rigid objects using their moments of inertia as it is determined with some examples below:



(a) Radius of Gyration of Uniform Thin rod

The moment of inertia of uniform thin rod of mass **M** and length **L** about an axis through its centre and perpendicular to its length is given by

$$I = \frac{1}{12} ML^2$$

If **k** is the radius of gyration of the rod about the axis, then we have

$$I = Mk^2$$

From the above equations, we have

$$Mk^2 = \frac{1}{12} ML^2$$

$$k = \frac{L\sqrt{3}}{6} \quad 1.11a$$

(b) Radius of Gyration of a Solid Sphere

The moment of inertia for a solid sphere of radius **R** and mass **M** is given by

$$I = \frac{2}{5} MR^2$$

If **k** is the radius of gyration of the solid sphere, then

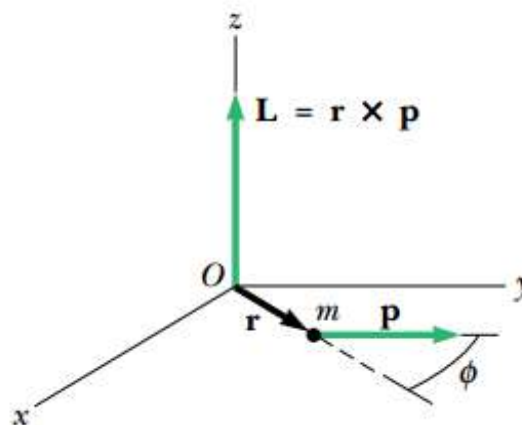
$$I = Mk^2$$

$$\text{Or, } Mk^2 = \frac{2}{5} MR^2$$

$$\text{So that, } k = R\sqrt{\frac{2}{5}} \quad 1.11b$$

1.4 ANGULAR MOMENTUM

The angular momentum \vec{L} of a particle of mass m moving along a circular path of radius r is defined as $\vec{L} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v})$ 1.12



Because $\mathbf{p} = m\mathbf{v}$, the magnitude of **L** is $\mathbf{L} = mvr \sin \phi$, where ϕ is the angle between **r** and **p**.

The S.I. unit of angular momentum is kilogram square metre per second (kgm^2s^{-1}).



Notice that the angular momentum for a particle moving in a circular path has been defined. Even a particle moving in a straight line has angular momentum about any axis displaced from the path of the particle.

A particle in uniform circular motion has a constant angular momentum about an axis through the centre of its path.

Torque causes the angular momentum L to change just as force causes linear momentum p to change, i.e., the torque acting on a particle is equal to the time rate of change of the particle's angular momentum.

The net external torque on a system of particles equals the time rate of change of angular momentum of the system

$$\sum \tau_{ext} = \frac{dL_{tot}}{dt} \quad 1.13$$

The angular momentum of a rotating rigid object is given by

$$\begin{aligned} \vec{L}_z &= \sum_i L_i = \sum_i m_i r_i^2 \omega = (\sum_i m_i r_i^2) \omega \\ \vec{L}_z &= I \omega \end{aligned} \quad 1.14$$

where we have recognized $\sum_i m_i r_i^2$ as the moment of inertia I of the object about the z axis.

The total angular momentum of a system is constant in both magnitude and direction if the resultant external torque acting on the system is zero, that is, if the system is isolated.

$$I_i \omega_i = I_f \omega_f = \text{constant}$$

Example 3: Estimate the magnitude of the angular momentum of a bowling ball with a mass of 6 kg and radius of 12 cm spinning at 10 rev/s.

$$I = \frac{2}{5} MR^2 = \frac{2}{5} (6.0 \text{ kg})(0.12 \text{ m})^2 = 0.035 \text{ kg} \cdot \text{m}^2$$

Therefore, the magnitude of the angular momentum is

$$\begin{aligned} L_z &= I \omega = (0.035 \text{ kg} \cdot \text{m}^2)(10 \text{ rev/s})(2\pi \text{ rad/rev}) \\ &= 2.2 \text{ kg} \cdot \text{m}^2 / \text{s} \end{aligned}$$

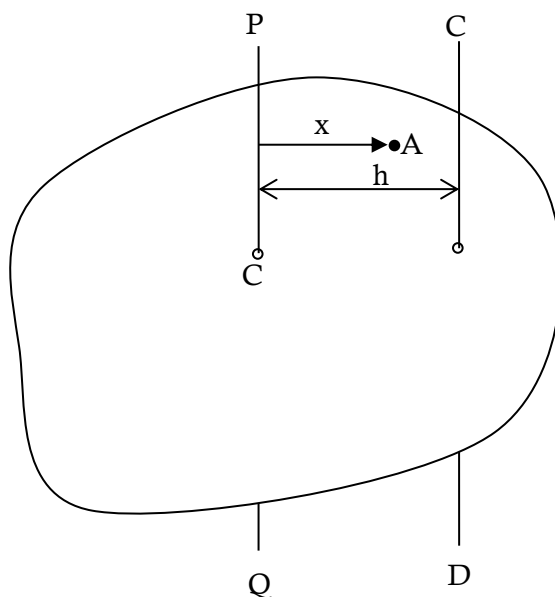
1.5 AXES THEOREM OF A RIGID BODY

The axes theorem responsible for the moment of inertia of a rigid body of mass M about an axis through its centre of gravity G when it rotates. These axes theorem, as enumerated below, will be treated with detail explanation as follows:



1.5.1 PARALLEL AXIS THEOREM

The moment of inertia of a body about any axis is equal to the moment of inertia I_G about a parallel axis through the centre of gravity of the body plus Mh^2 , where M is the mass of the body and h is the distance between the two axes, i.e., $I = I_G + Mh^2$.



Let I be the MI about CD

I_G be the MI about PQ, axis through point of the centre of gravity CG,

PQ is parallel to CD and h is perpendicular distance away from CD

Let a unit particle A of mass m be at distance x from PQ axis.

Moment of inertia I of A about CD = $m(h - x)^2$

i.e., $I = \sum m(h - x)^2$

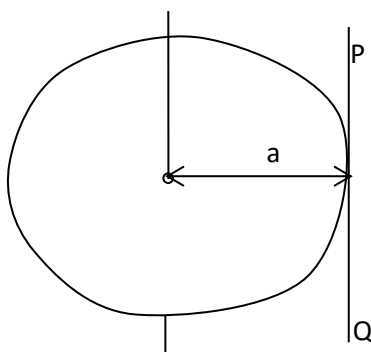
$$I = \sum mh^2 + \sum mx^2 + \sum 2mhx$$

But $\sum mh^2 = h^2 \sum m = Mh^2$ and $I_G = \sum mx^2$, where M is the total mass of the body.

Also, $\sum 2mhx = 2h \sum mx = 0$ since $\sum mx$ is the sum of moment about CG which is equal to 0. Using these simplified equations in combination, we have the parallel axis law proved:

$$I = I_G + Mh^2 \quad 1.15$$

Example 4: What is the moment of I of a disc, whose radius is 'a' and mass is m , about an axis PQ through a point on its circumference perpendicular to its plane (Its $I_G = \frac{1}{2}Ma^2$).





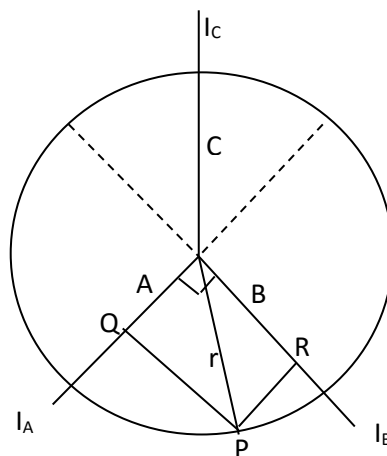
$I_{PQ} = I_G + Mh^2$ parallel axis theorem

So, $I = \frac{1}{2}Ma^2 + Ma^2$

i.e., $I_{PQ} = \frac{3}{2}Ma^2$

1.5.2 PERPENDICULAR AXIS THEOREM

The moment of inertia I of a body about any axis is the sum of the moment of inertia about any mutually perpendicular axes. This theorem is most useful when considering a body which is of regular form, i.e., symmetrical about two out of three axes. If the moment of inertia about these axes is mutually perpendicular to each other, then the third may be calculated.



Axes A and B are mutually perpendicular and are through symmetry of the body.

From the diagram, $I_C = \sum Mr^2$

But, $r^2 = (PQ)^2 + (PR)^2$

We now have the third one as

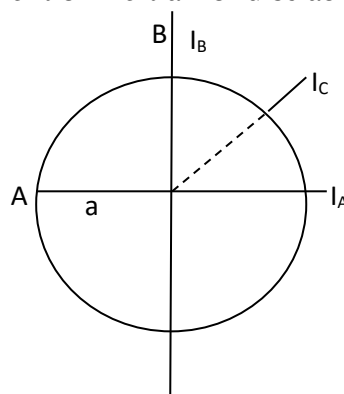
$$I_C = \sum M[(PQ)^2 + (PR)^2]$$

$$I_C = \sum M(PQ)^2 + \sum M(PR)^2$$

Using the simplified form, we have the perpendicular axis law proved:

$$I_C = I_A + I_B \quad 1.16$$

Example 5: Find the moment of inertia I of disc about axis through its diameter.





Using the perpendicular axis law expression:

$$I_C = I_A + I_B$$

where I_C = M.I. through its centre perpendicular to its plane, i.e., through its centre of mass.

$$\therefore I_C = \frac{1}{2} Mr^2$$

But, M.I. about A is $\frac{1}{2}$ of $\frac{1}{2} Mr^2$, and the rest half is M.I. about B.

Axes A and B are parallel to the plane of the disc.

$$\text{Thus, M.I. through diameter} = \frac{1}{4} Mr^2$$

1.6 KINETIC ENERGY OF A ROTATING RIGID BODY

If the mass of the i th particle is m_i and its tangential speed is v_i , its kinetic energy is $K_i = \frac{1}{2} m_i v_i^2$, but $v_i = r_i \omega$, thus the total kinetic energy is the sum of the kinetic energy due to all of the particles.

$$K_T = \sum_i K_i = \sum_i \frac{1}{2} m_i v_i^2 = \frac{1}{2} \sum_i m_i r_i^2 \omega^2$$

We can write this expression in the form

$$K_T = \frac{1}{2} (\sum_i m_i r_i^2) \omega^2 \quad 1.17$$

The quantity in parentheses is the moment of inertia given by: $I = \sum_i m_i r_i^2$.

Therefore, the kinetic energy of a rotating rigid body is

$$K_T = \frac{1}{2} I \omega^2 \quad 1.18$$

where I and ω are analogous to m and v in linear motion.

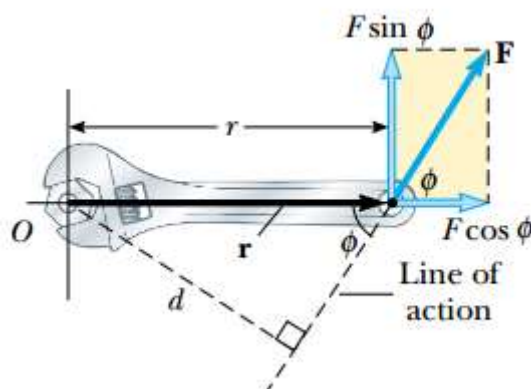
1.7 TORQUE $\vec{\tau}$

When a force is exerted on a rigid object pivoted about an axis, the object tends to rotate about that axis. The tendency of a force to rotate an object about some axis is measured by a vector quantity called **torque τ** .

Consider the wrench pivoted on the axis through O in Figure below. The applied force F acts at an angle ϕ to the horizontal. We define the magnitude of the torque associated with the force F by the expression

$$\vec{\tau} = rF \sin \phi = Fd \quad 1.19a$$

From the right triangle in Figure below that has the wrench as its hypotenuse, we see that $d = r \sin \phi$. The quantity d is called the **moment arm** (or lever arm) of F . Torque should not be confused with force. Torque has units of force times length, i.e., Newton-meters.



The torque acting on the particle is proportional to its angular acceleration, and the proportionality constant is the moment of inertia so that

$$\vec{\tau} = I\alpha \quad 1.19b$$

The vectorial notation of equation 9.17a is expressed in form of

$$\vec{\tau} = \vec{r} \times \vec{F} \quad 1.19c$$

1.8 WORK, ENERGY AND POWER OF A ROTATING RIGID BODY

The **power** delivered to a rotating rigid object (i.e. the rate at which work is done) by force F having torque τ and angular velocity ω is given by

$$P = \frac{dW}{dt} = \vec{\tau}\omega \quad 1.20$$

The total work done by the net external force acting on a rotating system where the angular displacement $d\theta$ changes from 0 to θ is given by:

$$W = \int_0^\theta \vec{\tau} d\theta = \vec{\tau}\theta \quad 1.21$$

That is, the work–kinetic energy theorem for rotational motion states that the net work done by external forces in rotating a symmetric rigid object about a fixed axis equals the change in the object's rotational energy.

Useful Equations in Rotational and Linear Motion

Rotational Motion About a Fixed Axis

Angular speed $\omega = d\theta/dt$

Angular acceleration $\alpha = d\omega/dt$

Net torque $\Sigma\tau = I\alpha$

If $\alpha = \text{constant}$
$$\begin{cases} \omega_f = \omega_i + \alpha t \\ \theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2 \\ \omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i) \end{cases}$$

Work $W = \int_{\theta_i}^{\theta_f} \tau d\theta$

Rotational kinetic energy $K_R = \frac{1}{2}I\omega^2$

Power $\mathcal{P} = \tau\omega$

Angular momentum $L = I\omega$

Net torque $\Sigma\tau = dL/dt$

Linear Motion

Linear speed $v = dx/dt$

Linear acceleration $a = dv/dt$

Net force $\Sigma F = ma$

If $a = \text{constant}$
$$\begin{cases} v_f = v_i + at \\ x_f = x_i + v_i t + \frac{1}{2}at^2 \\ v_f^2 = v_i^2 + 2a(x_f - x_i) \end{cases}$$

Work $W = \int_{x_i}^{x_f} F_x dx$

Kinetic energy $K = \frac{1}{2}mv^2$

Power $\mathcal{P} = Fv$

Linear momentum $p = mv$

Net force $\Sigma F = dp/dt$



EXERCISES

1. A diver makes 5 revolutions on the way from a 12 m high platform to the water. Assuming zero initial vertical velocity, find the diver's average angular velocity during the dive.
2. A body of mass 1.5 kg is whirled in a circle by a string of length 0.5 m. If the period of revolution is π sec. Calculate the force on the body by the string.
3. The SUG bus goes half-way around the round-about in 1 s. Calculate the bus's angular velocity.
4. If the mass of a bus is 5,000 kg and the distance between the bus and the centre of the round-about is 4 m, calculate the moment of inertia of the bus.
5. A flywheel of a gasoline engine is required to give up 300 J kinetic energy while its angular velocity decreases from 600 rev/min to 540 rev/min. What is the moment of inertia required?
6. Find the radial acceleration of an object at the equator due to the rotation of the earth.
7. An object of mass 4 kg moves round a circle of radius 6 m with a constant speed of 12 m/s. What is the angular speed of the object?
8. Compute the angular speed and angular acceleration of an automobile wheel 356 mm in radius when the car is moving at 72 km/h in 11 s.
9. A stone of mass 20 kg is whirled round in a vertical circular pattern by a rope of negligible mass of length 0.8 m. That is the moment of inertia of the stone?
10. A curve in a road forms part of a horizontal circle. As a car of mass 1,500 kg goes around it at constant speed 14.0 m/s, the total force on the driver has magnitude 130 N. What is the total vector force on the driver if the speed is 18.0 m/s instead?
11. A hawk flies in a horizontal arc of radius 12.0 m at a constant speed of 4.0 m/s. Find its centripetal acceleration.
12. A roller coaster car has a mass of 500 kg when fully loaded with passengers. (a) If the vehicle has a speed of 20.0 m/s at point A as it rolls at 10 m, what is the force exerted by the track on the car at this point? (b) What is the maximum speed the vehicle can have at B as it now rolls at 15 m and still remain on the track?
13. A small container of water is placed on a carousel inside a microwave oven, at a radius of 12.0 cm from the center. The turntable rotates steadily, turning through one revolution in each 7.25 s. What angle does the water surface make with the horizontal?
14. During a certain period of time, the angular position of a swinging door is described



by $\theta = 5.00 + 10.0t + 2.00t^2$, where θ is in radians and t is in seconds. Determine the angular position, angular speed, and angular acceleration of the door (a) at $t = 0$ (b) at $t = 3.0$ s.

15. A wheel starts from rest and rotates with constant angular acceleration to reach an angular speed of 12.0 rad/s in 3.0 s. Find (a) the magnitude of the angular acceleration of the wheel and (b) the angle in radians through which it rotates in this time.
16. A racing car travels on a circular track of radius 250 m . If the car moves with a constant linear speed of 45.0 m/s , find (a) its angular speed and (b) the magnitude and direction of its acceleration.
17. A uniform thin solid door has height 2.20 m , width 0.870 m , and mass 23.0 kg . Find its moment of inertia for rotation on its hinges.
18. A grind-wheel is started from rest given an angular acceleration of 3 rad/s^2 . Calculate its (a) angular velocity, and (b) angular displacement 8 s after starting to rotate.
19. Calculate the torque needed to bring a disc, which is rotating at 78 rev/min to a stop within two revolutions after the torque is applied. The disc has a moment of inertia $280 \text{ kg}\cdot\text{m}^2$.
20. A playground merry-go-round of radius $R = 2.0 \text{ m}$ has a moment of inertia $I = 250 \text{ kg}\cdot\text{m}^2$ and is rotating at 10.0 rev/min about a frictionless vertical axle. Facing the axle, a 25.0 kg child hops onto the merry-go-round and manages to sit down on the edge. What is the new angular speed of the merry-go-round?
21. A horizontal platform in the shape of a circular disk rotates freely in a horizontal plane about a frictionless vertical axle. The platform has a mass $M = 100 \text{ kg}$ and a radius $R = 2.0 \text{ m}$. A student whose mass is $m = 60 \text{ kg}$ walks slowly from the rim of the disk toward its centre. If the angular speed of the system is 2.0 rad/s when the student is at the rim, (a) what is the angular speed when he reaches a point $r = 0.50 \text{ m}$ from the centre? (b) What is the kinetic energy of the system before and after the student walks inward?
22. A uniform sphere with mass 28.0 kg and radius 0.380 m is rotating at constant angular velocity about a stationary axis that lies along a diameter of the sphere. If the kinetic energy of the sphere is 176 J , what is the tangential velocity of a point on the rim of the sphere?
23. One force acting on a machine part is $\vec{F} = (-5.0)\hat{i} + (4.0)\hat{j}$. The vector position from the origin to the point where the force is applied is $\vec{r} = (-0.45)\hat{i} + (0.15)\hat{j}$. (a) Determine the direction of the torque. (b) Calculate the vector torque for an axis at the origin produced by this force.
24. A playground merry-go-round has radius 2.40 m and moment of inertia $2100 \text{ kg}\cdot\text{m}^2$



about a vertical axle through its center, and it turns with negligible friction. (a) A child applies an 18.0 N force tangentially to the edge of the merry-go-round for 15.0 s. If the merry-go-round is initially at rest, what is its angular speed after this 15.0-s interval? (b) How much work did the child do on the merry-go-round? (c) What is the average power supplied by the child?

25. A 1.50 kg grinding wheel is in the form of a solid cylinder of radius 0.10 m. (a) What constant torque will bring it from rest to an angular speed 1200 *rev/min* in 2.5 s? (b) Through what angle has it turned during that time? (c) Calculate the work done by the torque. (d) What is the grinding wheel's kinetic energy when it is rotating at 1200 *rev/min*.

26. What is the moment of I of cylinder about an axis along its height through its centre perpendicular to its plane, if $I_G = Mr^2/2$?

27. Find the moment of inertia of lamina shown about axis AB through its centre if

$$I_{JK} = \frac{mb^2}{3} \text{ and } I_{PQ} = \frac{ma^2}{3}.$$

